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One-dimensional modeling of multiple-loop thermosyphons

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NOMENCLATURE

A, B	points common to all branches
c	specific heat
D	diameter of tubing
g_i'	gravitational acceleration in negative x_i direction
h	$= 4h'/\rho c D$
h'	heat transfer coefficient
L_i	length of branch from A to B
n	total number of branches
p_i	pressure
q_i	$= 4q_i'/\pi\rho c D^2$
q_i'	heat inflow per unit length
r	number of branches with known heat flux
s	number of identical branches
T_i	fluid temperature
T_{wi}	wall temperature
u_i	fluid velocity
x_i	longitudinal coordinate with origin at A.

Greek symbols

β	coefficient of volumetric expansion
λ	$= \lambda'/\rho c$
λ'	coefficient of thermal conductivity
ν	kinematic viscosity of fluid
ρ	density of fluid at reference temperature.

Subscripts

A	refers to point A
B	refers to point B
i	refers to branch i.

INTRODUCTION

THERMOSYPHONS are natural circulation systems in which density differences induced by heat transfer sustain the convective flow [1]. Heating modes represented by known heat flux as well as known wall temperature have been studied. A recent review of single loops has been made by Mertol and Greif [2]. In many practical applications, however, it is common to use multiple-loop thermosyphons, rather like a network. Most design calculations are made with the assumption of a single equivalent loop, disregarding possible differences between the branches. Recent work in this area includes refs. [3–6].

We shall consider the special situation in which n tubes each of arbitrary form start out from a single point A and finish at another point B. Of course other arrangements are possible. We shall also assume that all n branches consist of tubes of the same cross-sectional area but of different lengths L_i, $i = 1, \ldots, n$. A one-dimensional analysis will be used along with the Boussinesq approximation.

STEADY-STATE GOVERNING EQUATIONS

We take A to be the origin of longitudinal coordinates x_i, $i = 1, \ldots, n$ which run along each of the n branches. The arbitrary form of each one of the branches is indicated by the function $g_i'(x_i)$, $0 \leqslant x_i \leqslant L_i$, such that

$$\int_0^{L_i} g_i'(x_i)\,\mathrm{d}x_i$$

is the same for all branches.

Mass conservation at either A or B gives

$$\sum_{i=1}^{n} u_i = 0. \tag{1}$$

Though other power-law friction factors can be used, we shall assume frictional forces corresponding to Poiseuille flow in a straight pipe. The longitudinal momentum equation along branch i is given by [1]

$$-\frac{1}{\rho}\frac{\mathrm{d}p_i}{\mathrm{d}x_i} - \frac{32\nu}{D^2}u_i + \beta g_i' T_i - g_i'(1+\beta T_A) = 0. \tag{2}$$

The reference density and temperature are taken to be those at point A and are denoted by ρ and T_A, respectively. We can integrate equation (2), over the length of branch i to get

$$u_i L_i - \int_0^{L_i} \tilde{g}_i(x_i) T_i(x_i)\,\mathrm{d}x_i = a, \tag{3}$$

where

$$a = -\frac{D^2}{32\nu\rho}(p_B - p_A) - \frac{1}{\beta}(1+\beta T_A)\int_0^{L_i} \tilde{g}_i(x_i)\,\mathrm{d}x_i, \tag{4}$$

and

$$\tilde{g}_i(x_i) = \frac{\beta D^2}{32\nu} g_i'(x_i). \tag{5}$$

The energy equation will be written and integrated for heating modes represented by either known heat flux or known wall temperature along each branch.

(a) Known heat flux distribution $q_i'(x)$: The energy equation with axial conduction is

$$u_i \frac{\mathrm{d}T_i}{\mathrm{d}x_i} = q_i + \lambda \frac{\mathrm{d}^2 T_i}{\mathrm{d}x_i^2}. \tag{6}$$

The solution with boundary conditions $T_i = T_A$ at $x_i = 0$ and $T_i = T_B$ at $x_i = L_i$ is

$$\begin{aligned} T_i(x_i) &= F_{\lambda i}(x_i; T_A, T_B) \\ &= T_A + \frac{1}{\lambda}\int_0^{x_i} \exp\left(-\frac{u_i x_i''}{\lambda}\right)\int_0^{x_i''} \exp\left(\frac{u_i x_i'}{\lambda}\right) \\ &\quad \times q_i(x_i')\,\mathrm{d}x_i'\,\mathrm{d}x_i'' - \frac{\exp\left(-\dfrac{u_i x_i}{\lambda}\right) - 1}{\exp\left(-\dfrac{u_i L_i}{\lambda}\right) - 1}\Bigg\{(T_A - T_B) \\ &\quad + \frac{1}{\lambda}\int_0^{L_i} \exp\left(-\frac{u_i x_i''}{\lambda}\right) \\ &\quad \times \int_0^{x_i''} \exp\left(\frac{u_i x_i'}{\lambda}\right) q_i(x_i')\,\mathrm{d}x_i'\,\mathrm{d}x_i''\Bigg\}. \end{aligned} \tag{7}$$

If axial conduction is neglected the solution of equation (6), with the boundary condition $T_i = T_A$ at $x_i = 0$, is

$$T_i(x_i) = F_{0i}(x_i; T_A) = \frac{1}{u_i}\int_0^{x_i} q_i(x_i')\,dx_i' + T_A. \quad (8)$$

Of course, the boundary condition could have been applied at point B rather than point A. In general, the result would not be the same. This inconsistency in the non-conducting model leads to an overdetermined set of equations and is discussed in detail later.

(b) Known wall temperature distribution $T_{wi}(x_i)$: Using a constant heat transfer coefficient, the energy equation is

$$u_i\frac{dT_i}{dx_i} = h(T_{wi}-T_i)+\lambda\frac{d^2T_i}{dx_i^2}. \quad (9)$$

With the boundary conditions $T_i = T_A$ at $x_i = 0$ and $T_i = T_B$ at $x = L_i$, the solution is

$$T_i(x_i) = G_{\lambda i}(x_i; T_A, T_B) = C_1\,e^{r_1x_i}+C_2\,e^{r_2x_i}-\frac{h}{\lambda}e^{r_2x_i}\times\int_0^{x_i} e^{(r_1-r_2)x_i'}\int_0^{x_i'} e^{-r_1x_i''}\,T_{wi}(x_i'')\,dx_i''\,dx_i', \quad (10)$$

where

$$r_{1,2} = \frac{1}{2}\left\{\frac{u_i}{\lambda}\pm\sqrt{\frac{u_i^2}{\lambda^2}+\frac{4h}{\lambda}}\right\}.$$

C_1 and C_2 are constants which can be determined in terms of T_A and T_B from the boundary conditions.

Solution of energy equation (9) without axial conduction and with the boundary condition $T_i = T_A$ at $x_i = 0$, is

$$T_i(x_i) = G_{0i}(x_i; T_A) = e^{-hx_i/u_i}\left\{\frac{h}{u_i}\int_0^{x_i} e^{hx_i'/u_i}\,T_{wi}(x_i')\,dx_i'+T_A\right\}. \quad (11)$$

Again, $T_i = T_B$ at $x_i = L_i$ could have been used as a boundary condition.

At points A and B at which the branches meet, the total heat flux should sum to zero. Hence, at $x_i = 0$ and $x_i = L_i$ we have

$$\rho c\sum_{i=1}^{n} T_iu_i-\lambda\sum_{i=1}^{n}\frac{dT_i}{dx_i} = 0. \quad (12)$$

Since $T_i(0) = T_A$ and $T_i(L_i) = T_B$ for all i, this simplifies to

$$\sum_{i=1}^{n}\left.\frac{dT_i}{dx_i}\right|_{x_i=0} = \sum_{i=1}^{n}\left.\frac{dT_i}{dx_i}\right|_{x_i=L_i} = 0. \quad (13)$$

On neglecting axial conduction, the total heat flux towards A or B is due to convection alone. Thus only the first term in (12) is to be retained, and this is not independent of equation (1).

CONDUCTING MODEL

Of the n branches, the first r will be considered to have known heat fluxes while the rest will have known wall temperatures. The temperature distribution is

$$T_i(x_i) = \begin{cases} F_{\lambda i}(x_i; T_A, T_B), & i = 1,\ldots,r \\ G_{\lambda i}(x_i; T_A, T_B), & i = r+1,\ldots,n \end{cases} \quad (14)$$

with temperature conductions at both A and B being satisfied.

Substituting the temperature distribution (14) in (3) and (13) we have a set of equations in the $(n+3)$ unknowns u_i, a, T_A and T_B. The n equations in (3) along with two in (13) and one in (1) form the $(n+3)$ equations for this consistent set. Solutions are possible for any n and for any distributions of the known heat fluxes and/or known wall temperatures. The solutions are not necessarily unique. The equations are transcendental and numerical techniques have to be used for the determination of temperature distribution and fluid velocity in specific cases.

NON-CONDUCTING MODEL

Since most one-dimensional thermosyphon models neglect axial conduction we shall examine the consequences of such an assumption. The temperature is

$$T_i(x_i) = \begin{cases} F_{0i}(x_i; T_A), & i = 1,\ldots,r \\ G_{0i}(x_i; T_A), & i = r+1,\ldots,n \end{cases} \quad (15)$$

where $T_i = T_A$ at $x_i = 0$ is the only nodal condition satisfied. The temperature distribution (15) can be substituted in equation (3). Furthermore, due to continuity of fluid temperature at point B, we should have that

$$T_B = \begin{cases} F_{0i}(L_i; T_A), & i = 1,\ldots,r \\ G_{0i}(L_i; T_A), & i = r+1,\ldots,n. \end{cases} \quad (16)$$

We have $(2n+1)$ equations (1), (3) and (16) in the $(n+3)$ unknowns u_i, a, T_A and T_B. In general, a solution to the non-conducting model is thus *not* possible. Under the following special circumstances, however, solutions can be obtained:

(a) Single-loop thermosyphon: on taking $n = 2$, a set of five equations with five unknowns is obtained. The two branches starting from A and meeting at B are really parts of a single loop, for which both non-conducting as well as conducting models can be used [1, 2, 7].

(b) Identical branches: let us suppose that s of the n branches are identical in heating mode and distribution as well as in length and $g_i'(x_i)$ distribution. For the same velocity u_* in these s branches, we have $(2n-2s+3)$ independent equations, one from (1), $(n-s+1)$ from (3) and another $(n-s+1)$ from (16). There are $(n-s+4)$ unknowns including a, T_A, T_B and $(n-s+1)$ different velocities. Such a set of equations will have a solution if $n-s = 1$. On choosing s thus the multiple-loop thermosyphon is really simplified to a single-loop problem with all branches but one taken identical, which are being grouped together as a single branch.

CONCLUSIONS

The one-dimensional non-conducting equations have been used with success in single-loop problems. However, for the problem of n branches starting from one point and meeting at another, these equations do not form a consistent set in general. Two important special cases *can* be treated with both conductive as well as non-conductive models. These are the single-loop thermosyphon and the multiple-loop thermosyphon with all branches except one identical. It must be pointed out that though most multiple-loop analyses use the simplification of identical branches, this cannot be exactly so in reality. The effect of small differences between seemingly identical branches on the steady-state operation of the thermosyphon has yet to be fully determined. The limit of vanishing coefficient of thermal conductivity is also interesting. This is a singular problem and, in general, the $\lambda = 0$ limit cannot be obtained as $\lambda \to 0$.

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